# The Optical Quality of Shear Layers: Prediction and Improvement Thereof

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**Aerodynamic windows, which consist of one or more jets of high velocity gas, are frequently used to allow the extraction of laser beams from the laser cavity when the beam power density is sufficiently high to preclude the use of solid windows. Turbulent shear layers at the edges of the jets can produce random phase errors in the laser beam that can substantially reduce the maximum power density to which the beam can be focused. Improved semiempirical methods of predicting shear layer optical quality, applicable to both low and high Mach number shear layers, are presented and shown to give reasonably good agreement with one set of measurements on compressible shear layers. A number of recommendations for improving the optical quality of shear layers are presented. Some of these recommendations offer the potential for substantial improvements in optical quality.**

## Nomenclature

| | |
|---|---|
| $a$ | = constant, sound speed |
| $a_1$ | = sound speed of high-speed freestream or both freestreams |
| $A, b, B$ | = constants |
| $C_p$ | = specific heat at constant pressure |
| $h$ | $= C_p T = (\rho_a C_{p1} + \rho_b C_{p2}) T/\rho$ |
| $k$ | = wave number of coherent beam |
| $\ell$ | = mixing length, defined by Eq. (8) |
| $\ell_u$ | = mixing length, defined by $\overline{u'v'} = \ell_u^2 (\partial \bar{u}/\partial y)^2$ |
| $L$ | = slab thickness |
| $L_o$ | = $L$ for velocity matched layer |
| $L_{s1}, L_{s2}$ | = $L$ for first and second shear layers |
| $L_{50n,m}$ | = defined in Fig. 3 |
| $L_{95n}$ | = distance between the 5 and 95% points of the time-averaged refractive index profile across the shear layer |
| $M_s$ | $= \Delta u/a_1$ |
| $n$ | = refractive index |
| $n_{av}$ | $= (n_1 + n_2)/2$ |
| $R_s$ | = Strehl ratio |
| $s$ | $= \rho_a/\rho$ |
| $T$ | = temperature |
| $u$ | = velocity parallel to shear layer |
| $u_1, u_2, \ldots u_p$ | = velocities of freestreams |
| $v$ | = velocity normal to shear layer |
| $x$ | = distance from origin of shear layer |
| $y$ | = direction normal to shear layer |
| $\alpha$ | $= (\gamma - 1)/\gamma$ |
| $\beta$ | = Gladstone-Dale constant |
| $\gamma$ | = specific heat ratio |
| $\delta n$ | = refractive index change across shear layer |
| $\delta n_0$ | = $\delta n$ for velocity matched layer |
| $\delta n_{s1}, \delta n_{s2}$ | = $\delta n$ for first and second shear layers |
| $\delta u$ | = velocity difference across shear layer |
| $\delta_\omega$ | $= \Delta u/(\partial \bar{u}/\partial y)_{max}$ = vorticity thickness of shear layer |
| $\Delta n$ | $= n - \bar{n}$ = fluctuation in refractive index |
| $\Delta u$ | = velocity difference across shear layer |
| $\lambda_T$ | $= T_2/T_1$ |
| $\lambda_u$ | $= u_2/u_1$ |
| $\lambda_\alpha$ | $= \alpha_2/\alpha_1$ |
| $\lambda_\rho$ | $= \rho_2/\rho_1$ |
| $\Lambda$ | = integral scale length of fluctuation in $n$ |
| $\rho$ | = density |
| $\rho_a, \rho_b$ | = densities, at any point within the shear layer, of the components from the high- and low-speed freestreams, respectively |
| $\overline{\phi^2}$ | = mean square random phase error |
| $\langle(\ )\rangle$ | = time average |
| $\overline{(\ )}$ | = time average |
| $(\ )'$ | = fluctuation quantity, e.g., $u' = u - \bar{u}$ |

*Subscripts and Superscripts*

| | |
|---|---|
| *1* | = gas or conditions on high-speed side of shear layer |
| *2* | = gas or conditions on low-speed side of shear layer |

## I. Introduction

IN high-power density laser systems, the laser cavity pressure is often significantly different from the ambient value. The power densities may be sufficiently high to cause substantial heating of solid windows that might be used to allow extraction of the laser beam from the cavity. This heating can cause changes in the window refractive index and dimensional distortion that, in turn, can cause unacceptable phase errors to be impressed on the beam during passage through the window. Also, under some circumstances, catastrophic failure of solid windows under the laser beam heat load can occur. Under these conditions aerodynamic windows[1-4] are frequently used. In most aerodynamic windows a high-velocity gas jet flows roughly normal to the laser beam; the momentum of the jet is progressively deflected across the aperture of the window, supporting the pressure difference.

For aerodynamic windows an important consideration is the severity of the random phase errors introduced by the turbulent shear layers at the edges of the jets. Random phase errors are introduced because of refractive index differences across or within the shear layers. The random phase errors can greatly reduce the maximum power density to which the laser beam can be focussed, a severe disadvantage in many applications. For 10.6 $\mu$m laser beams traversing aerodynamic window systems, Refs. 2 and 4 indicate that a reduction in the maximum power density at a focus is ~5-10%, a loss that would not, in general, be serious. However, as the laser wavelength decreases, the reduction in the maximum power density at a focus becomes large very rapidly for the same gas flow geometry [see Eq. (3)].

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The present paper addresses the problem of random phase errors produced by turbulent shear layers in several ways. In Sec. II, a previously published[5] method of predicting the magnitude of the random phase errors produced by low and high Mach number shear layers is described. Under some conditions, the low Mach number method of Ref. 5 can yield fairly accurate results. Theoretical arguments are given which indicate that, under other conditions, it can be quite unsatisfactory. The high Mach number method of Ref. 5 requires the measurement of a suitably averaged value of $\overline{u'^2}/a^2$ for the shear layer, a fairly complex experimental undertaking. Improved semiempirical methods of predicting the random phase errors are presented here, requiring only knowledge of the time-averaged refractive index profile of the shear layer. The new methods are applicable to both low and high Mach number shear layers, and are shown to give predictions in reasonably good agreement with one set of measurements[6] made on high Mach number shear layers.

Section III presents a compilation of a number of methods of improving the optical quality of shear layers. One particular new result, presented in Sec. III.B and the Appendix, is a method of estimating the beam degradation to be expected on passage of a laser beam through a low-speed index-matched shear layer. The predicted degradation can be significant, contrary to what might naively be expected for such a shear layer, for wavelengths below 2 $\mu$m.

## II. Prediction of the Degradation of a Laser Beam by a Shear Layer

### A. Presentation of an Earlier Model

In Ref. 7 a coherent light beam is considered to propagate through a slab containing isotropic, homogeneous fluctuations in the refractive index. From Ref. 7 (p. 1740), it can be shown that the mean square random phase error of the beam on exiting the slab is given by

$$\overline{\phi^2} = 2k^2 \langle (\Delta n)^2 \rangle \Lambda L \tag{1}$$

provided that scintillation and diffraction effects can be neglected and that $\Lambda \ll L$. These provisions are probably fairly well satisfied for the shear layers of interest here, with $L$ taken to be a measure of the shear layer thickness.

An important measure of the degradation of a laser beam on traversing a region with random refractive index fluctuations is the peak intensity to which the beam can be focussed in the far field, divided by the corresponding value with no refractive index fluctuations present. This ratio is called the Strehl ratio and is given[8] by

$$R_s = 1 - \overline{\phi^2} = 1 - 2k^2 \langle (\Delta n)^2 \rangle \Lambda L \tag{2}$$

if $\overline{\phi^2}$ is small. If the distribution of the random phase errors at the slab exit is Gaussian, then[7]

$$R_s = \exp(-\overline{\phi^2}) = \exp(-2k^2 \langle (\Delta n)^2 \rangle \Lambda L) \tag{3}$$

for any value of $\overline{\phi^2}$.

Vu et al.[5] apply Eq. (2) to low Mach number shear layers and wakes as follows. $L$ is set equal to $\delta n / (\partial n/\partial y)_{\max}$. From the work of Batt,[9] Vu et al. take $\langle (\Delta n)^2 \rangle \approx (0.15\delta n)^2$ and $\Lambda \approx L/4$. With these substitutions, Eqs. (2) and (3) can be transformed to yield

$$1 - R_s = \overline{\phi^2} = 0.01125k^2 (\delta n)^2 L^2 \tag{4}$$

and

$$R_s = \exp(-\overline{\phi^2}) = \exp[-0.01125k^2 (\delta n)^2 L^2] \tag{5}$$

These results, apart from the generalization to $\overline{\phi^2}$ not small, are given in Ref. 5.

For high Mach number flows, Vu et al.[5] take compressibility effects into account by taking $\langle (\Delta n)^2 \rangle \approx (0.15\delta n)^2 + (n_{av} - 1)^2 (\overline{\rho'^2}/\rho^2)$ and taking $\overline{\rho'^2}/\rho^2 \approx \overline{u'^2}/a^2$.

For a laser beam traversing several shear layers in succession, assuming that the phase errors produced by each layer are independent of each other, we can extend Eq. (5) to read

$$R_s = \exp\{-0.01125k^2 \Sigma [(\delta n)^2 L^2]\} \tag{6}$$

### B. Development of New Model and Discussion

A shortcoming of the model presented in Sec. II.A is that $\langle (\Delta n)^2 \rangle$ and $\Lambda$, in general, vary through the shear layer. We make an assessment of the effect of this variation as follows. It is assumed that the refractive index fluctuations are locally homogeneous and isotropic, that $L \gg \Lambda$, and that the structure of the refractive index variations changes smoothly in the $y$ direction in the sense discussed in Tatarski.[10] The latter assumptions require that appreciable changes in $\langle (\Delta n)^2 \rangle$ and $\Lambda$ take place only over distances significantly larger than $\Lambda$. With these assumptions, we can extend Eq. (1) to the case of a shear layer with $\Lambda$ and $\langle (\Delta n)^2 \rangle$ varying parallel to the laser beam as follows:

$$\overline{\phi^2} = 2k^2 \int \langle (\Delta n)^2 \rangle \Lambda \mathrm{d}y \tag{7}$$

where the integration is through the shear layer.

We take $\Lambda/L = f_1(y/L)$, where $f_1$ is some function and $y$ is measured from a consistent origin (e.g., the point where the mean velocity is halfway between that of the two freestreams). We estimate $\langle (\Delta n)^2 \rangle$ using a simple mixing length argument

$$[\langle (\Delta n)^2 \rangle]^{1/2} = \frac{\ell \partial n}{\partial y} \tag{8}$$

We take $\ell = a\Lambda$, where $a$ is a constant and

$$\left(\frac{\partial n}{\partial y}\right)\left(\frac{L}{\delta n}\right) = f_2\left(\frac{y}{L}\right) \tag{9}$$

where $f_2$ is some function. Substituting $f_1$ and Eqs. (8) and (9) into Eq. (7) yields

$$\overline{\phi^2} = 2k^2 a^2 (\delta n)^2 L^2 \int f_1^3 (y/L) f_2^2 (y/L) \mathrm{d}(y/L) \tag{10}$$

One of the easiest ways to obtain information on the shear layer structure is to take time-averaged interferograms parallel to the shear layer. Assuming these to be the only data available, one then has information on $\partial n/\partial y$ but not on $\Lambda$. Using Eq. (10), the defining equations for $f_1$ and $f_2$, the relation $\ell = a\Lambda$, and taking $\Lambda$ to be a constant, one obtains

$$\overline{\phi^2} = 2k^2 \ell^3 a^{-1} \int \left(\frac{\partial n}{\partial y}\right)^2 \mathrm{d}y \tag{11}$$

If it is assumed that $\ell = bL_{95n}$, one obtains

$$\overline{\phi^2} = 2k^2 \left(\frac{b^3}{a}\right) L_{95n}^3 \int \left(\frac{\partial n}{\partial y}\right)^2 \mathrm{d}y \tag{12}$$

Predictions from the present model [Eqs. (11-13)] should be good for both low and high Mach number shear layers. Predictions of $\overline{\phi^2}$ obtained from Eq. (12) are now compared with experimental measurements[6] of $R_s$ for the shear layers produced by supersonic jets of four different gases. The experimental vaues of $\overline{\phi^2}$ are calculated using Eq. (3).

The gases used for the supersonic jets[6] were He, $N_2$, Ar, and 62% He/38% Ar. The jets had a Mach number of about 1.95 and discharged into still air. The jets were rectangular in cross section, and the optical path traversed two successive shear layers and was perpendicular to the shear layers. Further details are given in Ref. 6. Only data from Ref. 6

taken about 15 mm below the nozzle exit are used here; data taken close to the nozzle exit were complicated by the presence of Mach waves. From time-averaged interferograms taken parallel to the shear layer, $n$, $L$, $L_{95n}$, and $\partial n/\partial y$ were obtained. For the He/Ar jet, the refractive index was nearly matched across the shear layer and $L$ and $L_{95n}$ were not well defined. $L$ and $L_{95n}$ were plotted vs $\ln(\lambda_p)$ for the remaining three gases and interpolated values were used for the He/Ar jet in Eqs. (4) and (12).

$b^3/a$ was estimated as follows. In low-speed, constant-density shear layers, $\ell_u$ can be calculated using the method of Tollmein[11] and the more recent data presented by Brown and Roshko (Ref. 12, Fig. 10). $\ell_u$ is found to be approximately $0.119\delta_\omega$. Following the discussion of Hinze,[13] it is assumed that $\ell \simeq 2\ell_u$. From Vu et al.[5] and Batt,[9] $\Lambda \simeq L/4$. From Fiedler,[14] for low-speed, nearly constant-density shear layers, $L/\delta_\omega \simeq 1.36$ and $L_{95n}/\delta_\omega \simeq 1.31$. From these experimental results, for low-speed shear layers $a$ and $b^3/a$ are estimated to be equal to 0.70 and 0.0086, respectively. As a first approximation, this low-speed value of $b^3/a$ will be used to predict $\overline{\phi^2}$ for the shear layers studied in Ref. 6. Since the laser beam traverses two shear layers in the experimental setup of Ref. 6, the values of $\overline{\phi^2}$ predicted for a single shear layer using Eqs. (4) and (12) are doubled before being compared with those calculated from the measured values of $R_s$.

Figure 1 shows the comparison of the predicted and measured values of $\overline{\phi^2}$. The predictions using Eq. (12) are in reasonably good agreement with the experimental measurements. If $b^3/a$ is determined empirically as a function of shear layer Mach number, $b^3/a = 0.0126$ is found to give the best agreement between theory and experiment. With this value of $b^3/a$, the ratio of the predicted to the measured values of $\overline{\phi^2}$ ranges from about 0.7 to about 1.5 for the shear layers on Mach 1.95 jets. Predictions of $\overline{\phi^2}$ were also made using the low Mach number method of Ref. 5 [Eq. (4)] and were found to be quite unsatisfactory for these high Mach number shear layers. This is, of course, not unexpected. Predictions of $\overline{\phi^2}$ made using the high Mach number method of Ref. 5 might well be much better, but could not be made because of the unavailability of suitably averaged values of $\overline{u'^2}/a^2$ for the shear layers in question (see Sec. II.A).

It is not unexpected that the value of $b^3/a$ estimated from low-speed measurements (0.0086) does not give the best agreement between theory and experiment for the high-speed shear layers studied in Ref. 6. It seems preferable to determine $b^3/a$ for high-speed shear layers from experimental measurements of $R_s$, $L_{95n}$, and $\partial n/\partial y$. As we have seen, the experimental data of Ref. 6 indicate that $b^3/a = 0.0126$ is a fairly good value for the shear layers on Mach 1.95 jets discharging into still air. It would be desirable to have additional measurements of $R_s$, $L_{95n}$, and $\partial n/\partial y$ to allow empirical values of $b^3/a$ to be determined for a wider variety of shear layer conditions. However, as a first estimte for $\overline{\phi^2}$ for a shear layer or wake, when only the time-averaged refractive index profile across the layer is available, Eq. (12) with $b^3/a$ determined from Fig. 2 is suggested. $b^3/a$ in Fig. 2 for $M_s = 1.95$ is the value determined empirically from the data of Ref. 6; the value at $M_s = 0$ was estimated earlier from low Mach number data. The two data points are joined by a parabola since most compressibility effects vary as $M_s^2$ to a first approximation. More data defining $b^3/a = f(M_s)$ in Fig. 2 would be most welcome.

The following points should be noted when Eq. (12) and Fig. 2 are used. First, $\overline{\phi^2}$ predicted by Eq. (12) is only that due to turbulence in the shear layer or wake; contributions to $\overline{\phi^2}$ due to organized effects such as Mach waves and shear layer curvature must be computed separately and then added in. Second, $M_s$ should be calculated using the velocity difference across the shear layer and the sound speed of the high-speed stream (i.e., $M_s$ should be calculated as was done in Ref. 6). Third, when the shear layer is index matched or nearly index matched, $L_{95n}$ is not clearly defined; in such cases, $L_{50n,m}$, defined in Fig. 3, can be taken as an estimate for $L_{95n}$ in Eq. (12). We note that the value of $L_{95n}$ interpolated as described earlier for the He/Ar jet studied in Ref. 6 is equal to 1.09 times $L_{50n,m}$ for that jet. Finally, Eq. (12) and Fig. 3 are recommended only for the cases where the freestream turbulence levels are low and mixing is controlled by the velocity difference across the shear layers and the momentum defects in the wakes. In these cases, the assumption that $\ell = bL_{95n}$ should be reasonably good.

If data on both $\langle (\Delta n) \rangle^2$ and $\partial n/\partial y$ are available (e.g., see Ref. 15), some of the assumptions used in obtaining Eqs. (10) and (11) are unnecessary. In this case, using Eq. (7) and taking $\ell = a\Lambda$, we obtain

$$\overline{\phi^2} = 2k^2 a^{-1} \int \langle (\Delta n)^2 \rangle \ell \mathrm{d}y \tag{13}$$

which may, with $\ell$ given by Eq. (8), be used to predict $\overline{\phi^2}$. A difficulty with Eq. (13) is that, at the edges of the mixing layer, $\ell$ [calculated from the simple mixing length expression of Eq. (8)] is poorly defined and can diverge. Physically, the "true" mixing length ($\approx \Lambda$) cannot, of course, diverge, although it may increase somewhat near the edges of the mixing layer. It is recommended that the problem of the possible divergence of $\ell$ in Eq. (13) at the edges of the mixing layer be handled by limiting the maximum value of $\ell$ to two or

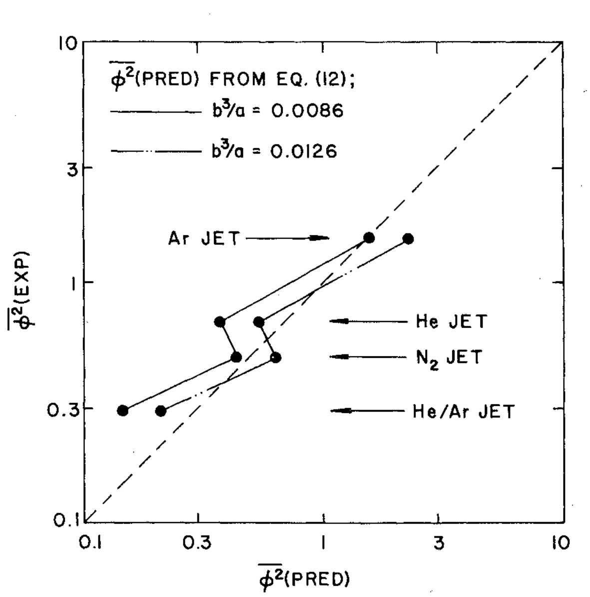


**Fig. 1 Comparison of experimental values of $\overline{\phi^2}$ obtained from measurements[6] of $R_s$ with predictions made using Eq. (12). Predictions are made using Eq. (12) for two values of $b^3/a$. The experimental measurements were made for Mach 1.95 jets of gases discharging into still air.**

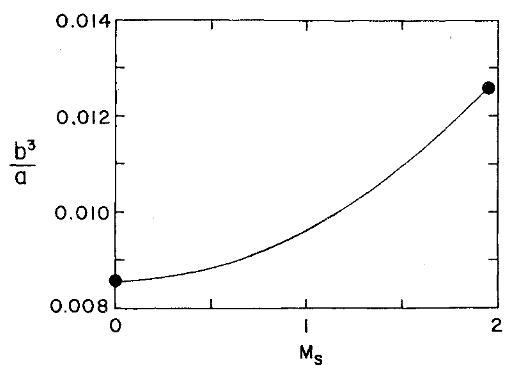


**Fig. 2 Suggested values of $b^3/a$ to be used with Eq. (12) for the prediction of the optical quality of shear layers. The abscissa is the shear layer Mach number $M_s$.**

three times the average value calculated in the interior regions in the shear layer. As long as the maximum allowed value of $\ell$ is large enough to permit $\ell$ variations in the interior regions of the shear layer to be followed, the integral in Eq. (13) is very insensitive to the exact maximum value chosen, unless this value is sufficiently large to allow totally unreasonable values of $\ell$ to occur in Eq. (13) near the edges of the shear layer.

The question can now be asked, for low Mach number shear layers, under what conditions can the simpler method of estimating $\overline{\phi^2}$, i.e., that of Eq. (4), be expected to yield satisfactory and unsatisfactory results? If $f_1$ and $f_2$ in Eq. (10) are universal functions for all low Mach number shear layers, then Eq. (10) will yield a result of the form of Eq. (4), even though $\langle(\Delta n)^2\rangle$ and $\Lambda$ are not taken to be constant through the shear layer. Obtaining satisfactory predictions using a correlation of the form of Eq. (4) will therefore depend upon the general validity of the relations $\Lambda = L \times$ (universal function of $y/L$) and $\partial n/\partial y = (\delta n/L) \times$ (universal function of $y/L$) for low Mach number shear layers.

A number of theoretical and experimental results[12,16-18] indicate that the properly normalized characteristics of shear layers, including the mean velocity and density profiles, depend relatively little on $\lambda_u$. Variations in $\lambda_u$ should, therefore, have little effect on the accuracy of correlations in the form of Eq. (4).

Substantial variations in $\lambda_\rho$ are known to produce significant changes in the normalized shape of the mean density profile, and hence would produce significant changes in the shape of $f_2(y/L)$ in Eq. (10). For example, low Mach number density profiles[9,10] for $\lambda_\rho \approx 1$ are very different in shape from those[12] for $\lambda_\rho = 7$. Konrad[15] gives low Mach number measurements of $\langle(\Delta n)^2\rangle$ and $\partial n/\partial y$ vs $y/L$ for $\lambda_u = 0.38$ and $\lambda_\rho = 1$ and 7. From these measurements, calculations of the ratio $R_\phi = \overline{\phi^2}(\lambda_\rho = 1)[\overline{\phi^2}(\lambda_\rho = 7)]^{-1}$ can be made using the value of $L$ and Eq. (4), the value of $L_{95n}$ and $\partial n/\partial y$ and Eq. (12), and the values of $\langle(\Delta n)^2\rangle$ and $\partial n/\partial y$ and Eqs. (13) and (8). In this case, there are no direct optical measurements of $\overline{\phi^2}$ against which to evaluate the estimation methods of Eqs. (4) and (12). However, the value of $R_\phi$ calculated using Eqs. (13) and (8) and $\langle(\Delta n)^2\rangle$ and $\partial n/\partial y$ may be used as a standard against which to judge the results of using Eqs. (4) and (12). $R_\phi$ calculated using Eq. (4) was 108% high, while the value calculated using Eq. (12) was 29% low. In the latter case, the accuracy of Eq. (4) was rather poor. $R_\phi$ found using Eq. (12) was not exact, but its accuracy was substantially superior to the value calculated using Eq. (4). The preceding discussion indicates that the use of Eq. (4) for shear layers with wide variations in $\lambda_\rho$ may result in substantial inaccuracies in the predicted values of $\overline{\phi^2}$.

The shape of the density and index profiles and $f_2$ in Eq. (10) can be substantially altered by the condition $\lambda_T \neq 1$, $\lambda_\alpha \neq 1$ in low Mach number shear layers (see Sec. III.C and Appendix). If these changes are significant, Eq. (4) may well yield inaccurate predictions. For example, the condition $\lambda_T \neq 1, \lambda_\alpha \neq 1$ can cause finite index gradients to appear within the shear layer even if $\delta n = 0$. Finite values of $\overline{\phi^2}$ would then be predicted by Eq. (12), while Eq. (4) would predict $\overline{\phi^2} = 0$.

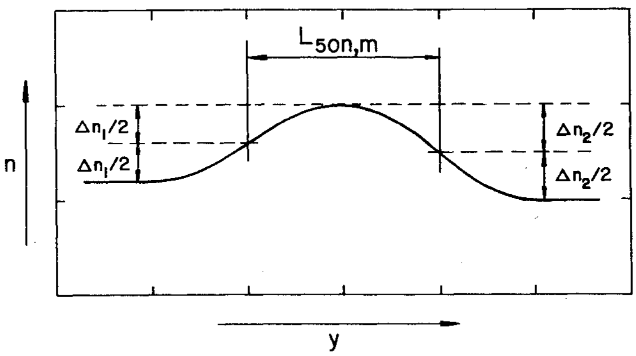


**Fig. 3 Profile of refractive index through a nearly index-matched shear layer, used to define $L_{50n,m}$.**

**Table 1 Recommendations for improving the optical quality of shear layers**

| Sec. III, Subsection | Flow(s) | Recommendation(s) |
|---|---|---|
| B | Shear layer, $\delta n \neq 0$ | Separate $\delta n$ and $\delta u$, using two shear layers[5] |
| C | Shear layer, $\delta n = 0$ | Minimize aerodynamic heating |
| | | Avoid the condition $\lambda_T \neq 1$, $\lambda_\alpha \neq 1$ |
| D | Shear layer, high speed, $\delta n = 0$ | Divide $\delta u$ between several layers |
| E | Wake, $\delta n \neq 0$ | Reduce momentum defect |
| F | Shear layer | Reduce or eliminate unfavorable curvature |
| G | Shear layer, wake | Use orifice plates upstream of layers |
| G | Shear layer, wake | Divide $\delta n$ between several layers[5] |
| G | Aerowindow | Use parallel flow geometry |

## III. Recommendations for Improving the Optical Quality of Shear Layers

### A. Table of Recommendations

Table 1 lists the recommendations discussed in this section. For any particular system configuration, practical considerations will frequently allow some of the recommendations given in Table 1 to be discarded quickly. For example, in many systems the pressure drop penalty across orifice plates will preclude their use. The recommendations of Table 1 are discussed individually in the following sections.

### B. Shear Layers with $\delta n \neq 0$: Separation of Velocity and Index Changes

In general, if there is a given shear layer in a system with index and velocity changes across it, the optical quality of the system can be improved by replacing the single shear layer by two shear layers, one taking the index change with zero velocity change and the other taking the velocity change with zero index change. Assuming that Eq. (4) applies, we can write, for the first case.

$$\overline{\phi_1^2} = 0.01125k^2(\delta n_{s1})^2 L_{s1}^2 \qquad (14)$$

and for the second,

$$\overline{\phi_2^2} = 0.01125k^2[(\delta n_{s2})^2 L_{s2}^2 + (\delta n_0)^2 L_0^2] \qquad (15)$$

Since $\delta n_{s2} = 0$, $\delta n_0 = \delta n_{s1}$, and $L_0$ is usually considerably smaller than $L_{s1}$, $\overline{\phi_2^2}$ will be significantly smaller than $\overline{\phi_1^2}$. Avidor[4] has shown experimentally that, for a subsonic aerodynamic window, $\overline{\phi_2^2}$ is substantially smaller than $\overline{\phi_1^2}$.

### C. Shear Layers with $\delta n = 0$

If there are no variations of index whatsoever in a shear layer, its optical quality should be perfect. One possible source of index variations in an index-matched shear layer is aerodynamic heating. For an index-matched shear layer with the same gas on both side, assuming the turbulent Prandtl number to be unity,[19] the maximum change in the mean refractive index in the shear layer can be estimated to be given by

$$\frac{(n_I - n)_{\max}}{n_I - I} = I - \left[I + \frac{\gamma - I}{8}\left(\frac{\delta u}{a_I}\right)^2\right]^{-I} \tag{16}$$

If $\delta u/a_I = 2$ and $\gamma = 1.4$, $(n_I - n)_{\max}/(n_I - 1)$ is equal to 0.17. Appreciable index variations can also occur within a low-speed index-matched shear layer if $\lambda_T$ *and* $\lambda_\alpha$ are both significantly different from unity (see Appendix). For a shear layer with $\lambda_T = 0.5$, $\gamma_I = 1.67$, and $\gamma_2 = 1.4$ so that $\lambda_\alpha = 1.404$, from Eq. (A5) $|(n_I - n)/(n_I - 1)|_{\max}$ is estimated to be 0.060.

These index variations can be significant, as the following examples show. An index-matched shear layer with $\lambda_u = 0$ and $\lambda_\rho = 1$ is considered. The gas on one side of the shear layer is taken to be room temperature air at standard pressure and the system aperture is taken to be 10 cm. $\overline{\phi^2}$ is estimated using Eq. (12) with $L_{95n}/\delta_\omega = 1.31$, $b^3/a = 0.0086$, and $L_{95n} \simeq L_{50n,m}$ (see Sec. II.B). The index profile is assumed to be a sinusoid with an amplitude of $|n_I - n_{\max}|/2$ and wavelength $2L_{50n,m}$. For the low-speed case, with $\lambda_T = 0.5$ and $\lambda_\alpha = 1.404$, we take[12] $\delta_\omega \simeq 0.18x$. $x$ is taken to be 5 cm. For the high-speed case, with $\delta u/a_I = 2$, $\delta_\omega$ is taken[12] to be equal to $0.09x$. This Mach number is representative[1-3] for supersonic aerodynamic windows. For various wavelengths of the optical beam, $R_s$ can then be estimated for Eqs. (3) and (12). See Table 2. For $\lambda > 2\mu$m, $R_s$ is estimated to be nearly unity, but for many shorter wavelengths of importance the beam degradation can be significant.

### D. Shear Layers with $\delta n = 0$ (High Speed)

If velocity differences high enough to cause appreciable aerodynamic heating are necessary at an index-matched interface, the optical quality of the system can be improved by changing from a single shear layer to a number of index-matched shear layers, each taking a part of the necessary total velocity difference. To simplify further discussion of this technique, it is assumed that the same gas is used for all freestream regions. We consider $p$ freestream regions, with velocities $u_I > u_2 > \ldots > u_{p-I} > u_p$. We assume, as in Sec. III.C, that $L_{95n} \simeq L_{50n,m}$ and that the index profile is a sinusoid with wavelength $2L_{50n,m}$. With these assumptions, Eq. (12) can be extended to the case where the beam traverses several shear layers and becomes

$$\overline{\phi^2} = k^2 \frac{\pi^2}{2}\left(\frac{b^3}{a}\right)\sum_{i=I}^{p}(n_I - n)^2_{\max,i}\, L^2_{50n,m,i} \tag{17}$$

where $(n_I - n)^2_{\max}$ is the maximum variation of $n$, squared, within the shear layer. We assume that $L_{50n,m} \propto \delta_\omega$ and take[12] $\delta_\omega \propto (1 - \lambda_u)/(I + \lambda_u)$. From Eq. (16), if $(n_I - n)_{\max}/(n_I - 1)$ is small, we may take $(n_I - n)_{\max} \propto (\delta u)^2$. Using these dependencies and Eq. (17), the ratio of $\overline{\phi^2}$ for a single index-matched shear layer to that for a number of index-matched shear layers taking the same total velocity difference can readily be found. Taking $u_p = 0$ and the optimum values for the velocities of the intermediate jets, it is found the $\overline{\phi^2}$ is decreased by factors of $\sim 20$ and $\sim 100$ by using two and three shear layers, respectively, instead of one shear layer.

In the preceding analysis, we have ignored the tendency of shear layer widths to decrease with increasing Mach number.[12] This works against the gains in optical quality that may be obtained by the recommended procedure. To get a feel for the severity of this effect, we consider a case with $u_p = 0$ and $(u_I - u_p)/a_I = 2$. For air jets discharging into stagnant air with equal freestream stagnation temperatures, shear layer widths are observed[12] to be roughly twice as small at Mach 2 as for low Mach number shear layers. We can, then, conservatively estimate the effect of compressibility on the recommended procedure by reducing the calculated width of the shear layer for the single-layer case by an extra factor of two relative to the calculated widths of the layers for multiple-shear-layer cases. With this correction, the factors by which $\overline{\phi^2}$ is decreased by using two and three layers instead of a single layer are reduced to $\sim 5$ and $\sim 25$, respectively. For the high-speed shear layer systems considered in Sec. III.C, use of two shear layers instead of one allows the estimated value of $R_s$ to be increased from 0.118 to 0.652 for the KrF laser ($\lambda = 0.248\mu$m) and from 0.594 to 0.901 for the HgBr laser ($\lambda = 0.502\mu$m).

**Table 2 Estimated values of Strehl ratio for laser beam traversing an index-matched shear layer**

| Beam wavelength $\lambda$ | Laser | $R_s$ for low-speed case $\lambda_T = 0.5$, $\lambda_\alpha = 1.404$ | $R_s$ for high-speed case, $\delta u/a_I = 2$ |
|---|---|---|---|
| 10.6 | $CO_2$ | 0.999 | 0.999 |
| 2.8 | HF | 0.992 | 0.983 |
| 1.3 | $I_2$ | 0.962 | 0.925 |
| 0.502 | HgBr | 0.771 | 0.594 |
| 0.248 | KrF | 0.345 | 0.118 |

### E. Wakes with Low Freestream Turbulence ($\delta n \neq 0$)

In this case, turbulent mixing (and consequent beam degradation) is due primarily to the presence of the momentum defect of the wake. Minimization or elimination of this momentum defect should allow significant improvements in optical quality to be obtained. Minimization of the nozzle length, splitter plate cooling, and conventional boundary-layer suction into the splitter plate could be used to reduce the wake momentum defect. In conventional geometries, none of these three approaches can reduce the momentum defect to zero. Suction at the very end of the splitter plate may permit the momentum defect to be eliminated entirely. (This technique has been suggested by Roshko's group at the California Institute of Technology.[20]) Such an approach seems less likely to be useful at high freestream velocities, since the ingested flow must negotiate at 180 deg turn. Blowing, both along the splitter plate and, in particular, at the end of the splitter plate, can reduce the wake momentum defect to zero and should remain useful at high freestream velocities.

### F. Effects of Curvature on Shear Layers

In free vortex aerodynamic windows the shear layers are curved. It is well known that curvature strongly affects the stability of boundary layers and shear layers.[21-23] The high-pressure shear layer in a free-vortex aerodynamic window has unfavorable curvature and, from empirical formulas in Bradshaw,[22] the mixing lengths can be estimated to be significantly increased by curvature effects in representative window configurations. Hence, the thickness of this shear layer could well be significantly increased by curvature effects. If the optical quality of an aerodynamic window were critical, it might be desirable to avoid free vortex windows and use window configurations in which the high-pressure shear layer had zero curvature.

### G. Other Recommendations

Another approach to handling wakes and shear layers with index differences is to install orifice plates at the beginning of the mixing region. Data have been taken[5,24] on the optical quality of wakes and shear layers with index differences using this technique. If the flow resistances of the orifice plates are sufficiently high, any splitter plate momentum defects upstream of the orifice plates will not be transmitted through the plates. The disadvantages of this technique are the production of turbulence throughout the freestreams and the pressure drop across the orifice plate. Vu et al.[5] discuss the effects of the scale and intensity of the freestream turbulence on the optical quality of the shear layers.

Division of a necessary total $\delta n$ between several shear layers can produce significant improvements in optical quality and is discussed in Ref. 5.

In some cases it may be worth considering the use of an aerodynamic window[25,26] with the flow parallel to the optical beam. Reference 26 gives some comparisons between this type of window and the more usual windows with the flow transverse to the optical beam. Under some conditions, the parallel flow window can, in principle, yield a much higher optical quality than the more usual transverse flow window designs.

## IV. Summary and Conclusions

The problem of random phase errors produced by turbulent shear layers extending across a laser beam is addressed. Theoretical arguments are presented which indicate that, under some conditions, an earlier method[5] of predicting the laser beam degradation across low Mach number turbulent shear layers can be significantly in error. Reference 5 also presents a method of high Mach number shear layers, which, however, requires the measurement of a suitably averaged value of $\overline{u'^2}/a^2$ for the shear layer, a measurement that can be fairly involved. A new, semiempirical method, requiring measurement of only the time-averaged refractive profile across the shear layer, is presented. This method is applicable to both low and high Mach number shear layers, and is shown to give reasonably good agreement with one series of beam quality measurements on supersonic flow shear layers. Recommendations for improving the optical quality of shear layers are presented. Some of the recommendations have the potential to allow substantial improvements in the optical quality of shear layers to be achieved. To the extent that this potential can be realized, it should be possible, while maintaining minimum beam degradation, to use shear layers or aerodynamic windows of increased aperture at a given wavelength and/or to use shear layers or windows of a given aperture at shorter wavelengths.

## Appendix

A shear layer or wake between dissimilar gases is considered. The following assumptions are made: 1) the Mach number is small, so that aerodynamic heating is negligible; 2) the mean pressure is uniform to first order throughout the flowfield; 3) molecular diffusion is neglected; and 4) the two gases are ideal with constant specific heats. We introduce $s=\rho_a/\rho$, $h=C_pT=(\rho_aC_{p1}+\rho_bC_{p2})T/\rho$, and $\rho=\rho_a+\rho_b$. At this point it is assumed that turbulent diffusion occurs in exactly the same manner for the two scalars, $s$ and $h$. In support of this assumption, it is noted that, in a low-speed shear layer, Batt[9] found the correlation coefficient between the temperature and concentration fluctuations to range between 0.90 and 0.95. If $s$ and $h$ diffuse in exactly the same manner, then, at any point within the shear layer, it can be shown that

$$h=As+B \tag{A1}$$

where $A$ and $B$ are constants. Using the boundary conditions at the edges of the shear layer to determine $A$ and $B$ and applying the ideal gas law, one may obtain the following expression for $\rho(s)$:

$$\frac{\rho}{\rho_1}=\frac{\lambda_\rho[(\lambda_\rho\lambda_T/\lambda_\alpha-1)s+1]}{[(\lambda_\rho\lambda_T-1)s+1][(\lambda_\rho/\lambda_\alpha-1)s+1]} \tag{A2}$$

The relation between the refractive index $n$ and $\rho$ and $s$ can be written

$$n-1=[(\beta_1-\beta_2)s+\beta_2]\rho \tag{A3}$$

From Eqs. (A2) and (A3), the refractive index can be uniquely related to the density at any point in the shear layer. Combining Eqs. (A2) and (A3), we obtain

$$n-1=\frac{[(\beta_1-\beta_2)s+\beta_2]\rho_2[(\lambda_\rho\lambda_T/\lambda_\alpha-1)s+1]}{[(\lambda_\rho\lambda_T-1)s+1][(\lambda_\rho/\lambda_\alpha-1)s+1]} \tag{A4}$$

Finally, for the index-matched case, $n_1-1=\beta_1\rho_1=n_2-1=\beta_2\rho_2$ and Eq. (A4) can be written

$$n-1=(n_1-1)\frac{[(\lambda_\rho-1)s+1][(\lambda_\rho\lambda_T/\lambda_\alpha-1)s+1]}{[(\lambda_\rho\lambda_T-1)s+1][(\lambda_\rho/\lambda_\alpha-1)s+1]} \tag{A5}$$

From Eq. (A5), it is noted that if $\lambda_T=1$ or $\lambda_\alpha=1$, $n-1$ is constant throughout the shear layer. If both $\lambda_T\neq1$ and $\lambda_\alpha\neq1$, $n-1$ is not constant across the shear layer. In this case, the maximum or minimum value of $(n-n_1)/(n_1-1)$ occurring with the shear layer can easily be obtained using Eq. (A5). Figure A1 shows the maximum or minimum values of $(n-n_1)/(n_1-1)$ for a low-speed index-matched shear layer plotted as a function of $\lambda_T$ and $\lambda_\alpha$. These values turn out to be independent of $\lambda_\rho$, even though $\lambda_\rho$ appears in Eq. (A5).

Although the preceding analysis is simple and certainly not exact, it does allow a valuable estimate to be obtained for the amount of refractive index variation to be expected within a low-speed index-matched shear layer with $\lambda_T\neq1$ and $\lambda_\alpha\neq1$.

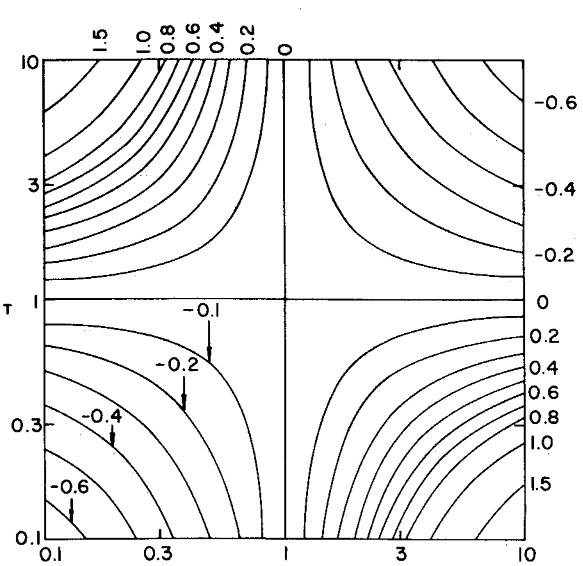


**Fig. A1 Predicted maximum or minimum values of $(n-n_1)/(n_1-1)$ within a low-speed index-matched shear layer as a function of $\lambda_T$ and $\lambda_\alpha$.**


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